Triangle Similarity Criteria

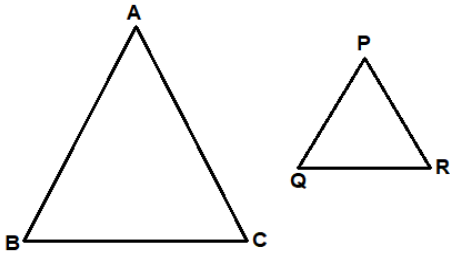
All congruent figures are similar, but it does not mean that all similar figures are congruent.

Two polygons of the same number of sides are similar, if:

a. Their corresponding angles are equal.
b. Their corresponding sides are in the same ratio.

Two triangles are similar, if:

a. Their corresponding angles are equal.
b. Their corresponding sides are in the same ratio.



$\triangle ABC \sim \triangle PQR$

By C.P.S.T.

Corresponding Parts of Similar triangles

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

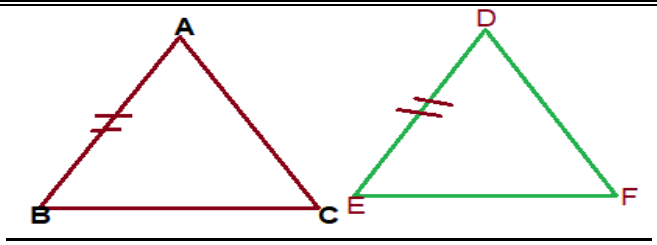
Theorem 6.3: (AAA similarity)

If the corresponding angles of the two triangles are the same, the corresponding sides are in the same ratio. So they are similar triangles.

Given: In $\triangle ABC$ and $\triangle DEF$, $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$.

To Prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ Hence $\triangle ABC \sim \triangle DEF$

Case I When AB = DE



Proof: - In ⊿ABC and ⊿DEF,

∠A = ∠D (Given)

∠B = ∠E(Given)

AB = DE (Supposed)

⊿ABC ≅ ⊿DEF (By A-S-A criteria)

AB = DE, BC = EF, AC = DF (BY CPCT)

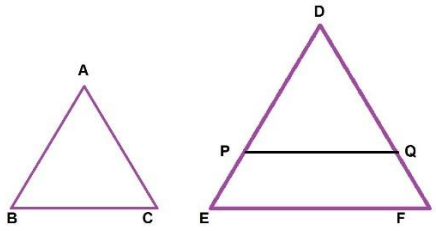
$\frac{AB}{DE} = 1, \frac{BC}{EF} = 1, \frac{AC}{DF} = 1$

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Hence⊿ABC ~⊿DEF Proved

Case II When AB ≠ DE, AB< DE

Construction: In ⊿DEF, take ponts P and Q in sides DE and DF such that AB = DP and AC = DQ; join P__Q.



Proof:

In ΔABC and ΔDEF.

Here, AB = DP and AC = DQ (by Construction)

∠A = ∠D (Given)

So, ΔABC ≅ ΔDPQ. (by S-A-S AXIOM)

So, ∠ABC = ∠DPQ,

but as given ∠ABC = ∠DEF.

This means that ∠DPQ = ∠DEF. [Corresponding angles]

Hence, PQ ∥ EF

DP/PE = DQ/DF (Parallel line divides the sides of the triangle in equal ratio)

Hence, AB/DE = BC/EF = AC/DF.

Hence⊿ABC ~⊿DEF Proved